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CHALLENGES OF MODERN CONTROL THEORY

Richard Bellman

PREPARED FOR:

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The RAND Corporation
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The RAND Corporation
1700 MAIN ST • SANTA MONICA • CALIFORNIA

PREFACE

Part of the RAND research program consists of basic supporting studies in mathematics. The present Memorandum outlines challenging problems of modern control theory, with special emphasis on the biomedical domain.

SUMMARY

The fundamental objective of the descriptive theory which embraces most of classical science is that of determining the future history of a system given sufficient information concerning the present state. The fundamental objective of the new scientific discipline called "control theory" is that of modifying the behavior of a system subject to various constraints of feasibility so as to achieve desired aims.

From the mathematical point of view, the prime purpose is to approximate to reality by means of hierarchies of mathematical models, each representing a projection of the scientific scene. In the present RAND Memorandum, the author discusses some aspects of this attitude and briefly indicates the possible contributions of modern control theory to the biomedical domain.

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CHALLENGES OF MODERN CONTROL THEORY

1. MODERN CONTROL THEORY

The fundamental objective of the descriptive theory which embraces most of classical science is that of determining the future history of a system given sufficient information concerning the present state. The fundamental objective of the new scientific discipline called "control theory" is that of modifying the behavior of a system subject to various constraints of feasibility so as to achieve desired aims [1].

If all aspects of the system under consideration are known and understood, the determination of optimal control policies may still be a matter of considerable difficulty. If, as is almost invariably the case in practice, major uncertainties exist, then the difficulties are magnified to such an extent as to pose mathematical and scientific challenges of the first magnitude.

It is reasonable to postulate that the basic task of the engineer is to achieve partial control with partial understanding, limited objectives with limited means. As scientific knowledge accrues and mathematical techniques improve, we can expect to achieve firmer control.

From the mathematical point of view, therefore, the prime purpose is to approximate to reality by means of hierarchies of mathematical models, each representing a projection of the scientific scene. In what follows, we shall discuss some aspects of this attitude. Finally, we shall briefly indicate the possible contributions of modern control theory to the biomedical domains. If, indeed, engineering is science in the service of man, then this is the region of greatest benefit to mankind.

2. PROBLEM SOLVING

To solve problems is the proper function of the scientific intellectual. Problem solving is, however, no simple process [2], consisting, as it does, of such clearly distinguishable phases as recognition, formulation, analysis, computation, comparison, and reformulation. Since these phases are interlinked, we see that this is an adaptive process of multistage type [3].

In the following sections, we shall consider some of these categories and indicate how this convenient nomenclature conceals some extremely difficult and yet quite rewarding problems.

3. RECOGNITION

It is essential, for the mathematician concerned with using his skills to help in the solution of problems of the physical world, that he be able to recognize those matters which are susceptible to the collection of devices, tricks, and legerdemain called mathematics, and those which are not.

This is a pattern-recognition problem, and is no less difficult to treat than most other specimens of this genre. There are certain key features which alert the trained observer to the possibility of using the techniques of modern control theory. Some of these features are the following:

(a) Decision-making.
(b) Multistage processes, repetition over time, recurrence.
(c) Complexity and uncertainty.
(d) Questions of communication and use of data (information theory in the broader, more meaningful sense).
(e) Questions of storage and retrieval of data; diagnosis.
(f) Memory.
(g) Approximation methods.

Classifications of this type are useful. Nevertheless, recognition of significant research areas is best taught by example rather than precept, and, even more efficiently, by pointing out obvious dead ends, mined-out areas, morasses, and bogs in current research.

It is rather sad to realize that despite the existence of so many fascinating problem areas all about, many, and indeed most, students in mathematics and engineering are directed into dull and trivial research areas, and given only the bluntest of instruments to work with.

4. FORMULATION

A small set of carefully chosen problems in control theory can be posed in terms of the calculus of variations. As far as the further development of the theory is concerned, this is most unfortunate. Having the full machinery of a powerful and elegant mathematical theory at their disposal to treat special problems is apt to make many people forget that they need entirely different methods to treat different and, in general, more important aspects of general control theory.

On the whole, classical techniques are ineffectual as far as decision making in the face of complexity and uncertainty is concerned. In the formulation of a control process, it is necessary to specify state variables, cause-and-effect relations, policies, and criteria.

It is essential to recognize the many degrees of freedom we possess, since each degree opens up a new avenue of approximation techniques, e.g., "approximation in policy space" and "stochastic approximation" [1].

Above all, the formulation must be intimately correlated to the computation, and, in particular, to calculation using digital and analogue computers. It

should be kept in mind that most of traditional mathematical analysis is either unconcerned with numerical matters or else oriented toward hand computation. In any case, the computational technology of the twentieth century deserves better than the analysis of the eighteenth or, at best, nineteenth century.

5. ANALYSIS AND COMPUTATION

The line between analysis and computation is not a sharp one. Let us attempt to illustrate this most essential point. By the solution of an equation, we mean the exhibition of an algorithm for producing the desired values, individual numbers or functional values. Once upon a time, the solution of

$$u' = u, \quad u(0) = 1, \tag{5.1}$$

in the form $u = e^x$ was considered a solution. Nowadays, if we are interested in obtaining numerical values of the function e^x, we return to (5.1), if we are using either a digital or an analogue computer. In other words, which is the problem and which is the solution depends on a number of factors. "Problem" and "solution" are relative, not absolute terms.

The point we wish to make is that the construction of an algorithm for the determination of optimal control using a digital computer may in many cases be most conveniently accomplished using analytic and computational approaches quite different from those currently presented in the textbooks, and even in the research journals. The theories of dynamic programming [1] and invariant imbedding [4] are devoted to the general aim of exploiting the structure of a process in order to calculate the desired information in an efficient manner.

Many processes have been analysed using these theories, and a number of new algorithms and approximation

techniques obtained. Nevertheless, the problem area of the classification and exploitation of the structure of processes has been little appreciated and less explored. Here lie opportunities galore for the hardy pioneer. Let us mention in this context the work of Kron [5].

It would be eminently desirable to understand in various precise senses what is meant by "levels of complexity" and "hierarchies of uncertainty."

6. CONTROL THEORY IN BIOLOGY AND MEDICINE

The direct applications of control theory to biology and medicine are immediate, following the ideas expressed in Wiener [6] or Grodins [7]. Diseases of the nervous system such as Parkinson's disease, of the brain such as cerebral palsey, prosthetics to alleviate the loss of limbs, deafness, blindness, cancer, heart trouble, mental health—all of these fit so clearly into the general domain of modern control theory as to need no elaboration.

Consider, for example, the cancer problem. A group of cells accustomed to carrying out a specified set of functions in the body suddenly disobey orders and priorities and eventually result in the destruction of the entire system. Were we to understand the causes for this aberrant behavior, we could conceivably take more effective steps in the early detection and subsequent control of cancer.

We have emphasized before that the engineering attitude is partial control with partial understanding. Is it not possible, and even probable, that an engineering approach to cancer will improve the present procedures? We do not expect a solution on these terms, but a decrease in mortality of even five per cent a year would be eminently satisfying.

Let us emphasize that it is the attitude that distinguishes engineering, not the tools. It is thus entirely reasonable to expect that those individuals who have had vast experience in dealing with complexity and uncertainty in one part of the scientific domain will do well in another part requiring the same skills.

The need for control theory in biology and medicine is clear, multiply clear; the intellectual challenge is there; only the motivation appears to be missing in the thousands of brilliant scientists who devote all of their skill, time, and energy to space travel, nuclear physics, and other activities for the glory of Science. Let us hope that this self-imposed asceticism will not persist indefinitely and that most of these Stylites will eventually clamber down from their lofty perches and help us here on earth with our human problems.

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